Assignment-2

Q1. Find Solution using:-

a) NWCM

b) LCM

c) Vogel's Approximation Method:

	D_1	D_2	D_3	D_4	Supply
S_1	11	13	17	14	250
S_2	16	18	14	10	300
S_3	21	24	13	10	400
Demand	200	225	275	250	

a) NWCM:

	D_1	D_2	D_3	D_4	Supply
S_1	11 200	13 50	17	14	250-50-0
S_2	16	18 175	14 125	10	300-125-0
S_3	21	24	13 150	10 250	400-250-0
Demand	200-0	225-175-0	275-150-0	250-0	

Cost:- $(11 \times 200) + (13 \times 50) + (18 \times 175) + (14 \times 125) + (13 \times 150) + (10 \times 250)$

$= 12200$

Allocated Cells = $C = m + n - 1$

$C = 3 + 4 - 1 = 6$

$\boxed{C = 6}$

b) Least Cost:-

	D_1	D_2	D_3	D_4	Supply
S_1	11 [200]	13 [50]	17	14	250
S_2	10	18 [125]	14 [125]	10	300 175 0
S_3	21	24	13 [125]	10 [175]	300 175 0
Demand	200 0	225 175 0	225 125 0	175 0	

$\therefore$ Demand = Supply = 950

min trans cost:-

$= 11(200) + 13(50) + 18(175) + 14(125) + 13(125) + 10(175)$

$= 12200$

No. of allocated cell $= C = m + n - 1$

$\Rightarrow 6 = 3 + 4 - 1$

$\boxed{6 = 6}$

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c) Vogels Method:

	D_1	D_2	D_3	D_4	Supply	Row		
S_1	11	13	17	14	250	2	-	1
S_2	16	18	14	10	300	4	4	4
S_3	21	24	13	10	400	3	3	3
D.	200	225	225	125				
	5	5	1	0				
		5	1	0				
		6	1	0				
			1	0				
			13	10				

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min cost:

$11 \times 200 + 13 \times 50 + 13 \times 225 + 10 \times 125 + 13 \times 125 + 10 \times 125$

$\Rightarrow 12075$

Allocated cells: $6 = m+n-1$

$\Rightarrow 6 = 6$

Q2. Find sol. using Vogel's approximation method, also find optimal soln using modi method

	D_1	D_2	D_3	D_4	Supply	Row	Penalty			
S_1	19 [5]	30	50	10 [2]	7	9	9	40	40	
S_2	70	30	40 [7]	60 [2]	9	10	20	20	20	10
S_3	40	8 [8]	70	20 [10]	18	12	20	5	-	
D.	5	8	7	14						

Col.	21	22	10	10
Penalty	21	-	10	10
			10	10
			10	50
			40	60

	D_1	D_2	D_3	D_4	Supply
S_1	19 [5]	30	50	10 [2]	7
S_2	70	30	40 [7]	60 [2]	9
S_3	40	8 [8]	70	20 [10]	18
D.	5	8	7	14	

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min. transport. cost:

$19 \times 5 + 8 \times 8 + 40 \times 7 + 10 \times 2 + 60 \times 2 + 20 \times 10$

$= 729$

No of alloc. $= 6 = m + n - 1$

$= 6 = 6$

optimality test using modi Method.

Iteration : 1 :

find u_i and v_j for all occupied cells

$u_1 = 0$

$c_{11} = u_1 + v_1 = v_1 = c_{11} - u_1 = 19 - 0 = 19 = 19$

$c_{14} = u_1 + v_4 = v_4 = c_{14} - u_1 = 10 - 0 = 10 = 10$

$c_{23} = u_2 + v_3 = v_3 = c_{23} - u_2 = 40 - 50 = -10$

$c_{24} = u_2 + v_4 = v_2 = c_{24} - v_4 = 60 - 10 = 50$

$c_{32} = u_3 + v_2 = v_2 = c_{32} - u_3 = 8 - 10 = -2$

$c_{34} = u_3 + v_4 = v_3 = c_{34} - v_4 = 20 - 10 = 10$

	D_1	D_2	D_3	D_4		Supply
S_1	19 (5)	30	50	10	$u_1 = 0$	7
S_2	70	30	40	60 (2)	$v_2 = 50$	9
S_3	40	8 (10)	70	20 (10)	$v_3 = 10$	18
D_4	5	8	7	4		
	$v_1 = 19$	$v_2 = -2$	$v_3 = -10$	$v_4 = 10$		

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for unoccupied cells :

$d_{12} = v_1 + v_2 - c_{12} = 0 + (-2) - 30 = -32$

$d_{13} = v_1 + v_3 - c_{13} = 0 - 10 - 50 = -60$

$d_{21} = v_2 + v_1 - c_{21} = 50 + 19 - 70 = -1$

$d_{22} = v_2 + v_2 - c_{22} = 50 + (-2) - 30 = 18$

$d_{31} = v_3 + v_1 - c_{31} = 10 + 19 - 40 = -11$

$d_{33} = v_3 + v_3 - c_{33} = 10 + (-10) - 70 = -70$

	D_1	D_2	D_3	D_4	Supply	
S_1	19	30	50	10	7	$v_1 = 0$
S_2	70	30	40	60	9	$v_2 = 32$
S_3	40	8	70	20	18	$v_3 = -10$
D.	5	8	7	4		
	$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$		

Iter. II for occupied cells.

$v_1 = 0$

$c_{11} = v_1 + v_1 = v_1 = c_{11} - v_1 = 19 - 0 = 19$

$c_{22} = v_2 + v_2 = v_2 = c_{22} - v_2 = 30 - (-2) = 32$

$c_{14} = v_1 + v_4 = v_4 = c_{14} - v_1 = 10 - 0 = 10$

$c_{23} = v_2 + v_3 = v_3 = c_{23} - v_2 = 40 - 32 = 8$

$c_{32} = v_3 + v_2 = v_2 = c_{32} - v_3 = 8 - 10 = -2$

$c_{34} = v_3 + v_4 = v_3 = c_{34} - v_4 = 20 - 10 = 10$

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for Unoccupied cells:

$d_{12} = v_1 + v_2 - c_{12} = 0 - 2 - 30 = -32$

$d_{13} = v_1 + v_3 - c_{13} = 10 + 8 - 50 = -42$

$d_{24} = v_2 + v_4 - c_{24} = 32 + 10 - 60 = -18$

$d_{31} = v_3 + v_1 - c_{31} = 10 + 19 - 40 = -11$

$d_{33} = v_3 + v_3 - c_{33} = 10 + 8 - 70 = -52$

$d_{41} = v_2 + v_4 - c_{24} = 32 + 19 - 70 = -19$

Since all $d_{ij} \leq 0$ final optimal soln:

	D_1	D_2	D_3	D_4	Supply
S_1	19 (5)	30	50	10 (2)	7
S_2	70	30 (9)	40 (7)	60	9
S_3	40	8 (8)	70	20 (12)	18
D.	5	8	7	4	

Transp. Cost :-

$19+5+10+2+3\times2+4\times0+6\times12 = 143$

Q. Find I.B using Northwest Corner Method (NWCM) also find optimal sol^n using MODI method.

	D_1	D_2	D_3	D_4	S_i
S_1	11 (200)	13 (50)	12	14	250
S_2	16	19 (175)	14 (125)	10	300
S_3	21	24	13 (150)	10 (100)	400
D_j	200	225	275	250	
	0	150	125	0	

Total allocation $ min. cost :-

$11\times200 + 13\times50 + 19\times175 + 14\times125 + 13\times150 + 10\times100$

$= 10,250$

optimality using MODI method for finding u_i & v_j occupied cells.

$v_1 = 0$

$c_{11} \; v_1 + u_1 = u_1 = c_{11} - v_1 = 11 - 0 = 11$

$u_1 = v_2 + u_2 = v_2 = c_{12} - u_1 = 13 - 11 = 2$

$c_{22} = v_2 + u_2 = u_2 = c_{22} - v_2 = 19 - 2 = 17$

$c_{33} = v_3 + u_3 = u_3 = c_{33} - v_3 = 13 - 9 = 4$

$c_{34} = v_3 + u_4 = u_4 = c_{34} - v_3 = 10 - 4 = 6$

	D_1	D_2	D_3	D_4	Supply	
S_1	11 (200)	13 (150)	17	14	250	$u_1 = 0$
S_2	16	18 (125)	14 (125)	10 (125)	300	$u_2 = 5$
S_3	21	24	13 (100)	10 (200)	400	$v_3 = 4$
D	200	225	275	250		
	$v_1 = 11$	$v_2 = 13$	$v_3 = 9$	$v_4 = 6$		

Calculation for unoccupied cells:

$d_{13} = u_1 + v_3 - c_{13} = 0 + 9 - 12 = 9$

$d_{14} = u_1 + v_4 - c_{14} = 0 + 6 - 14 = -8$

$d_{21} = u_2 + v_1 - c_{21} = 5 + 11 - 16 = 0$

$d_{24} = u_2 + v_4 - c_{24} = 5 + 6 - 10 = 1$

$d_{31} = u_3 + v_1 - c_{31} = 4 + 11 - 21 = -6$

$d_{32} = u_3 + v_2 - c_{32} = 4 + 13 - 24 = -7$

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					Supply	
	11 (200)	13 (50)	17	14	250	$u_1 = 0$
	16	18 (125)	14	10 (125)	300	$u_2 = 5$
	21	24	13	10 (175)	400	$v_3 = 5$
D.	200	225	275	250		
	$v_1 = 11$	$v_2 = 13$	$v_3 = 9$	$v_4 = 5$		

Iteration = II

For occupied cells:

$u_1 = 0$

$c_{11} = u_1 + v_1 = v_1 = c_{11} - u_1 = 11 - 0 = 11$

$c_{12} = u_1 + v_2 = v_2 = c_{12} - u_1 = 13 - 0 = 13$

$c_{22} = u_2 + v_2 = v_2 = c_{22} - v_2 = 18 - 13 = 5$

$c_{24} = u_2 + v_4 = v_4 = c_{24} - v_2 = 10 - 5 = 5$

$c_{33} = u_3 + v_3 = v_3 = c_{33} - u_3 = 13 - 5 = 8$

$c_{34} = u_3 + v_4 = v_3 = c_{34} - v_4 = 10 - 5 = 5$

for unoccupied cells:

$d_{13} = v_1 + v_3 - c_{13} = 0 + 8 - 17 = -9$

$d_{14} = v_1 + v_4 - c_{14} = 0 + 5 - 14 = -9$

$d_{21} = v_2 + v_1 - c_{21} = 5 + 11 - 16 = 0$

$d_{23} = v_2 + v_3 - c_{23} = 5 + 8 - 14 = -1$

$d_{31} = v_3 + v_1 - c_{31} = 5 + 11 - 21 = -5$

$d_{32} = v_3 + v_2 - c_{32} = 5 + 15 - 24 = -4$

Optimum soln achieve :- dij 20

	D_1	D_2	D_3	D_4	Supply
S_1	11(200)	13(50)	17	14	250
S_2	16	18(175)	14	10(125)	300
S_3	21	24	13(275)	10(125)	400
Demand	200	225	275	250	

∴ min cost :- $11(200) + 13(50) + 18(175) + 10(125) + 13(275) + 10(125)$

$= 12075$

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Q4. Find Soln of Assignment problem using hungarian method.

Workload	I	II	III	IV	V
A	10	5	13	15	16
B	3	7	12	13	6
C	10	7	8	8	2
D	7	11	9	7	12
E	7	9	10	4	12

Row min.

	I	II	III	IV	V
A	5	0	3	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

Col. min.

5	0	0	10	11
0	6	15	10	3
8	5	0	0	0
0	4	2	0	5
3	5	6	0	8

Row & Col. scanning

5	[0]	0	10	11
[0]	6	15	10	3
8	5	[0]	0	0
0	4	2	[0]	5
3	5	6	0	8

Job → Allocation.

	I	II	III	IV	V
A	5	0	6	10	9
B	0	6	13	10	1
C	10	7	0	2	0
D	0	4	0	0	3
E	3	5	4	0	6

Again Row ✓ Col. Scanning

5	1	0	6	10	9
0		6	13	10	1
10		7	0	2	0
0		4	0	0	3
3		5	4	0	6

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Job = Alloc.

Job	alloc.	Profit
I	B	5
II	A	5
III	B	9
IV	E	4
V	C	2
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Q5. Find Soln of Assign. Prob. using hungarian method (min)

Job	I	II	III	IV
A	9	14	19	15
B	7	17	20	18
C	9	18	21	18
D	10	12	18	18
E	10	15	21	16

Here Job ≠ citing

Row min.

Work	I	II	III	IV	V
A	9	14	19	15	0
B	7	17	20	19	0
C	9	18	21	18	0
D	10	12	18	19	0
E	10	15	21	16	0

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Col min.

Job.	I	II	III	IV	V
A	2	2	1	0	0
B	0	5	2	4	0
C	2	6	3	3	0
D	5	5	0	4	0
E	3	3	3	1	0

Job ≠ allocation

2	2	1	0	1
0	5	2	4	1
2	5	2	2	0
3	0	0	4	1
2	2	2	0	0

Row Sum & Colm Sum.

2	2	1	0	1
0	5	2	4	1
1	5	2	2	0
3	0	0	4	1
2	2	2	0	0

Job ≠ allocation

0	4	0	0	1
1	4	1	4	1
4	4	1	2	0
0	0	0	5	2

2	1	0	0	1
0	4	1	4	1
1	4	1	2	0
4	0	0	5	2
2	1	1	0	0

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∴ Job = allocation = 5

Job	allocation	profit
A	VI	19
B	I	7
C	V	0
D	II	12
E	IV	16
		54

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Ex: Max. Case :- find Soln using hungarian meth

work	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

Minimized Cube

0	7	14	21
12	17	22	27
12	17	22	27
18	22	26	30

Row Minimal

0	7	14	21
0	5	10	15
0	5	10	15
0	4	8	12

Col. min.

0	3	6	9
0	1	2	3
0	1	2	3
0	0	0	0

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Row & Column Scanning

	I	II	III	IV
A	0	3	C	9
B	0	1	2	3
C	0	1	2	3
D	0	0	0	0

Job ≠ allocation

0	2	5	8
0	0	1	2
0	0	1	2
1	0	0	0

Job ≠ allocation

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0	2	4	7
0	0	0	1
0	0	0	1
2	1	0	0

Job ≠ allocation

Signal method :-

0	2	4	7
0	0	0	1
0	0	0	1
2	1	0	0

∴ Job = allocation

I	A	42
II	B	25
III	C	20
IV	D	12
		= [illegible] X

Q7. Travelling Salesman Problem using hungarian method. Find Soln of travelling salesman problem given below.

Work	I	II	III	IV
1	x	4	9	5
2	6	x	4	8
3	9	4	x	9
4	5	3	9	x

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Row min.

x	0	5	1
2	x	0	4
5	0	x	5
0	3	4	x

Col. min.

x	0	5	0
2	x	0	3
5	0	x	4
0	3	4	x

Row → Colum. Scanning

Work	I	II	III	IV
1	X	0		[0]
2	2	X	[0]	3
3	5	[0]	X	4
4	[0]	3	4	X

Jobs = allocations

1 - IV 2 - 3

4 - I 3 - 2

3-2-
2-3
1-4

1-2 + 3-4 = 0+4 = 4

1-3 + 2-4 = 5+3 = 8

original :-

1-4 = 5

2-3 = 4

3-2 = 4

4-4 = 5

18

∴ Smallest path

Total cost

4 + original path = 4+18

= 22